Sediment Transport (in Estuaries)
Modes of Sediment Transport

An incomplete introduction to:

• Bed Load
• Suspended Load
• Instrumentation and Challenges
Rouse Parameter, $P$

Suspended load:

\[ \frac{w_s}{\kappa U_\star} \leq 0.8 \]

Bedload:

\[ \frac{w_s}{\kappa U_\star} > 2 \]

Open University, 1999
Figure 6.8 The Shields diagram, as modified by Vanoni (1964). To find the shear stress required to move a given sediment, calculate

\[
\frac{(d_s/\nu)}{\sqrt{0.1(\gamma_s/\gamma - 1)}}g d_s
\]

Locate this value on the scale given, find the intersection of the Shields curve, and read off the value of \( \beta \) from the ordinate. (In this figure \( d_s \) is the sediment grain size.)

Middleton and Southard 1984
Consider steady horizontally uniform flow.

Estimate stress at the bed:

**Law of the Wall**

\[ \frac{u}{u_*} = \frac{1}{\kappa} \ln \left( \frac{z}{z_0} \right) \]

**Drag Coefficient**

\[ \tau_b = \rho C_D u_{100}^2 \]

**Reynolds Stresses**

\[ \tau = -\rho u' w' \]

\( u_z, u_{100}, u' w' \rightarrow u_*, \tau_0 \rightarrow \tau_c \rightarrow \text{suspended load} \)

\( \rightarrow \text{bed load} \)
How to find $w_s$?

A force balance leads to Stokes Law

\[ F_D = C_D \rho \frac{U^2}{2} A \]

\[ F_g = \frac{\pi}{6} d^3 (\rho_s - \rho) g \]

\[ C_D = \frac{24}{Re} \]

\[ \text{Re} = \frac{ud}{v} \]

\[ v = \frac{\mu}{\rho} \]

\[ u = w_s = \frac{(\rho_s - \rho)}{18 \mu} gd^2 \]

$F_D$ = drag force

$F_g$ = gravitational force

$C_D$ = drag coefficient

$\rho, \rho_s$ = density of water, sediment

$A$ = area of sphere

$d$ = diameter of sphere

$g$ = acceleration due to gravity
Bed Load

Rouse Parameter

\[ \frac{W_s}{k u_*} > 2 \]

\[ q_b = \frac{8[\Psi(t) - \Psi_{cr}]^2}{(\rho_s - \rho)g} \]

Recall critical Shield’s parameter:

After Meyer-Peter and Muller (see Wright 1995)

\[ \Psi_{cr} = \frac{\tau_{cr}}{g D_s \rho (\frac{\rho_s}{\rho} - 1)} \]

Wright 1995
Bedload

$q_b \propto u^3$

$q_b =$ bedload transport rate

Rate of bedload depends on power, i.e. rate of flow doing work on sediment

Energy $\sim$ velocity$^2$
Powers $=$ energy $\times$ velocity

Power $\sim$ velocity$^3$

Can lead to fractionation and sorting by size, transport of different sizes at different phases of tide, net fluxes

Open University 1999
Bedforms
Result of bedload, tight feedback, affect partitioning of form drag and skin friction

Relatively low Froude numbers, eventually as Fr increases, bedforms wash out, create plain bed

Open University 1999
Instrumentation:
Fan beam, imaging sonar and poking eyeball

Sherwood, Rubin, USGS
Rouse Parameter, $P$

Suspended load:

\[ \frac{w_s}{\kappa u_*} < 0.8 \]

Bedload:

\[ \frac{w_s}{\kappa u_*} > 2 \]
• Suspended Sediment Transport Equation (Sediment Mass Balance)

\[ \frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x_i} = \frac{\partial}{\partial x_i} \left( K_s \frac{\partial C}{\partial x_i} \right) - \frac{\partial}{\partial z} (w_s C) \]

advection mixing settling

\[ U, \text{ flow velocity } U(x,y,z,t) \]
\[ C, \text{ susp sed conc } C(z,C_n) \]
\[ K_s, \text{ eddy diffusivity, } K_s(\text{ht above bed, } z, \text{boundary sheer stress, } \tau_b, U_* \text{roughness, } k_s \]
\[ \text{stratification } \partial \rho/\partial z, \rho(S,T,C) \]
\[ w_s, \text{ particle settling velocity } w_s(\text{diameter } D, \text{density, } \rho, \rho_s) \]

Need to measure flow, fluid, sediment
Suspended Sediment Transport

\[
\frac{\partial C}{\partial t} = \frac{\partial}{\partial z} \left( K_s \frac{\partial C}{\partial z} \right) + u \frac{\partial C}{\partial x} + w_s \frac{\partial C}{\partial z}
\]

1: Entrainment/Mixing
2: Advection
3: Deposition by Settling

\[-K_s \frac{\partial C}{\partial z} = w_s C\]

\[K_s = \beta ku_* z(1 - z/h)\]

\[C = C_a \text{ at } z=z_a\]
Rouse Equation: Vertical Distribution of Suspended Sediments

\[
\frac{C_z}{C_a} = \left( \frac{h - z}{h - z_a} \right) \left( \frac{z_a}{\kappa u_*} \right)^{w_s} 
\]

- \( C_z \): concentration at height above bed \( z \)
- \( C_a \): concentration at height above bed \( a \)
- \( h \): total water depth
- \( w_s \): settling velocity
- \( u_* \): shear velocity
- \( \kappa \): von Karman constant, 0.41

Fig. 1. Dimensionless vertical profiles of suspended sediment (Rouse profiles) for various values of the Rouse parameter \( Ro = w_s / \kappa u_* \). The magnitude of the suspended sediment is scaled by the near-bed concentration, which depends on the bed conditions and \( u_* \).

Geyer 1993
\[
\frac{C_z}{C_a} = \left( \frac{h-z}{z_a} \right) \frac{z_a}{h-z_a} \frac{w_s}{\kappa U_*}
\]
What do we need to know?

\[
\frac{C_z}{C_a} = \left(\frac{h-z}{z} \bullet \frac{z_a}{h-z_a}\right)^{\frac{w_s}{\kappa u_*}}
\]

- \(C_z\) = conc at ht above bed \(z\)
- \(C_a\) = conc at ht above bed \(a\)
- \(h\) = total water depth
- \(w_s\) = settling velocity
- \(u_*\) = shear velocity
- \(\kappa\) = von karman constant, 0.41

Fig. 1. Dimensionless vertical profiles of suspended sediment (Rouse profiles) for various values of the Rouse parameter \(R_o = w_s/\beta u_*\). The magnitude of the suspended sediment is scaled by the near-bed concentration, which depends on the bed conditions and \(u_*\).

Geyer 1993
Shear Velocity (forcing)

Law of the Wall
\[ \frac{u}{u_*} = \frac{1}{\kappa} \ln \left( \frac{z}{z_0} \right) \]

Drag Coefficient
\[ \tau_b = \rho C_D u_{100}^2 \]

Reynolds Stresses
\[ \tau = -\rho u' \overline{w}' \]
Eddy Diffusivity $K_s$ function of:

- boundary sheer stress, $\tau_b$, $U_*$ due to currents (commonly measured by $\partial u/\partial z$ or $\langle u'w' \rangle$) AND waves
- Also roughness, stratification

\[
\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x_i} = \frac{\partial}{\partial x_i} \left( K_s \frac{\partial C}{\partial x_i} \right) - \frac{\partial}{\partial z} \left( w_s C \right)
\]
\[
\frac{\partial C}{\partial t} + \mathbf{U} \frac{\partial C}{\partial x_i} = \frac{\partial}{\partial x_i} \left( K_s \frac{\partial C}{\partial x_i} \right) - \frac{\partial}{\partial z} (w_s C)
\]

**Flow Measurements (Eulerian):**

At a point:
- Mechanical Rotors
- Electromagnetic Current Meters
- Acoustic Doppler Velocimeter

Through the water column:
**Acoustic Doppler Current Profiler**

**Variables:**
- \( U \), flow velocity \( U(x,y,z,t) \)
- \( C \), suspended sediment concentration \( C(z,C_n) \)
- \( K_s \), eddy diffusivity
- \( K_s \) (height above bed, \( z \),
  boundary shear stress, \( \tau_b \), \( U_* \)
  roughness, \( k_s \)
  stratification \( \frac{\partial \rho}{\partial z}, \rho(S,T,C) \)
- \( w_s \), particle settling velocity
  \( w_s \) (diameter \( d \), density, \( \rho \), \( \rho_s \))
Settling velocity - a big challenge for cohesive sediments

In the marine environment fine particles interact to flocculate, aggregate, agglomerate

Floc pictures from Tim Milligan
Wash load

\[ w_s \approx \text{fine sand} \]

Figure 2-41. Schematic sequence of typical structures and size of flocs and floc groups; adapted from McDowell and O'Connor (1977). Reproduced by permission of John Wiley and Sons.
Cohesive Sediments

- Clay particles \( d < 2 \text{um} \), plate-like
- Surfaces have charges, electrostatic forces
- Comparable to gravity
- Clay particles stick together (cohesion, stickiness)
- Becomes significant when 5-10% clay by weight
• Flocculation, aggregation, agglomeration
• Face of platelet generally –ve charge
• In salt water double layer is suppressed
• Electrostatic charge repulses $V_R \propto 1/e^R$
• Van der Waals attractive $V_A \propto 1/R^2$
• Fresh water: repulsive dominates
• Salt water: reduces surface charge, $V_A$ dominates
• Flocculation begins 0.6-2.4 psu
• Organic matter enhances
• Flocculation less effective as $T \uparrow$
How to bring particles together?

1. Brownian motion
   collision rate of frequency

   \[ K_B = \frac{4KTN}{3\mu} \]

   \( K \) = Boltzmann's constant
   \( T \) = temperature
   \( n \) = number of particles
   \( \mu \) = molecular viscosity

   What is it a function of?

Maybe effective at high conc of small particles (\(d<0.5\mu\)), otherwise considered insignificant.
2. Shear

$$K_v = \frac{4}{3} n R^3 \frac{du}{dz}$$

$R = \text{sum of radii}$

$\frac{du}{dz} = \text{velocity gradient}$

Effective with large particles at small shear, or small particles at large shear

$K_v > K_B$
3. Differential Settling

\[ K_s \alpha \pi R^2 \Delta wn \]

\[ R = \text{sum of radii} \]

\[ \Delta w = \text{relative settling vel} \]

Not very effective
4. Encounters due to turbulent shear

\[ K_i = 1.294R^3 \left( \frac{\varepsilon}{\nu} \right)^{\frac{1}{2}} n_1 n_2 \]

\[ \varepsilon = \text{turbulent dissipation rate} \]

Effect of Turbulence

What is it a function of?
Fig. 4.13 Settling flux as a function of sediment concentration for Lake Okeechobee (Florida) mud. Data points are from Hwang (1989). The curve is based on (4.19).

Figure 5. Settling velocity and settling flux as functions of concentration.

Ross and Mehta, 1989
Flocs have odd shapes, grow and break up with changing concentration and shear, and have variable density.

What do you use for settling velocity?
\[
\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x_i} = \frac{\partial}{\partial x_i} \left( K_s \frac{\partial C}{\partial x_i} \right) - \frac{\partial}{\partial z} \left( w_s C \right)
\]

\( w_s \), particle settling velocity
\( w_s \) (diameter \( D \), density \( \rho \), \( \rho_s \), \( \rho_f \))

- Stokes calculation for sphere (problematic for fine sediments)
- Estimate from in situ images and assumed density
- Measure directly
- Lots of examples of optical, laser, settling, capture and bottom withdrawal

Milligan
INSSECT
IN situ Size and SEttling Column Tripod

- Digital floc camera
- MAVS
- Recovery package
- OBS
- Fins
- Video Camera & settling column
- Sediment trap
- Rotating frame

Thanks Mikkelsen, Milligan, Hill
Try to model:
See Winterwerp and van Kesteren 2004 for description of flocculation model based on fractal structure of flocs. Can determine equilibrium floc diameter, $D_f$, and differential (excess) density, $\Delta \rho_f$.

Balance between gravitational and drag forces:

\[
F_g = \alpha \frac{\pi}{6} D_f^3 g \Delta \rho_f \quad \quad \quad \quad \quad F_d = \beta C_D \frac{1}{2} \rho_w \frac{\pi}{4} D_f^2 w_{s,f}^2
\]

\[
C_D = \frac{24}{Re_p} (1 + 0.15 Re_p^{0.687}) \quad \quad \quad \quad \text{Based on empirical data}
\]

\[
w_{s,f} = \frac{\alpha}{18 \beta} \left( \rho_s - \rho_w \right) g D_p^{3-n_f} \frac{D_f^{n_f-1}}{1 + 0.15 Re_f^{0.687}}
\]

$\alpha$ and $\beta$ are coefficients depending on shape (sphericity)

$n_f$ is the fractal dimension varying from $\sim 1.4$ (marine snow) to 1.7-2.2 for estuarine and coastal waters. $n_f=2$ yields good results for the Ems estuary

$D_f$, $D_p$ are diameter of floc and primary particle

For $\alpha=\beta=1$ and $n_f=3$ you get Stokes
Winterwerp and van Kesteren 2004

Results in pretty typical values. See Hill 1998 and other work by Hill, Milligan, Fox, and others.

In practice...
For fine silt and clay fraction one assumes a floc fraction and assigns a floc settling velocity of ~1 mm/sec.
Reference Concentration, $C_a$

- Measure directly
- Calculated from excess shear stress
- Erosion rate

Latter two are function of bed characteristics
Reference concentration, $C_a$, for size class $j$ of bed sediment, $C_b$, a function of excess shear stress, $\theta$. $\gamma$ is an empirical constant.

$$C_{a,j} = C_{b,j} \frac{\gamma \theta_j}{1 + \gamma \theta_j}$$

$$\theta_j = \frac{\tau - \tau_{cr,j}}{\tau_{cr}}$$

Wright 1995
Challenges:
For cohesive sediments $\tau_c$ varies with depth (and therefore time)

Winterwerp and van Kesteren 2004

Dyer 1986
\[
\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x_i} = \frac{\partial}{\partial x_i} \left( K_s \frac{\partial C}{\partial x_i} \right) - \frac{\partial}{\partial z} (w C)
\]

Suspended Sediment Concentration

At a point:
Discrete samples with a bottle or pump, filter and weigh

At a point, continuous in time, (quasi-instantaneous depth profile):
Optical (e.g. transmissometer, OBS)

Through the water column:
Acoustic (e.g. ABS (near bottom), ADCP)

Fine sediments: changing particle characteristics

\( U \), flow velocity \( U(x,y,z,t) \)
\( C \), susp sed conc \( C(z,C_n) \)
\( K_s \), eddy diffusivity,
\( K_s \) (ht above bed, \( z \),
  boundary shear stress, \( \tau_b \), \( U_* \)
  roughness, \( k_s \)
  stratification \( \partial \rho/\partial z, \rho(S,T,C) \))
\( w_s \), particle settling velocity
\( w_s \) (diameter \( d \), density, \( \rho, \rho_s \))
Profiling Tripod

- Salinity
- Temperature
- Pressure
- Current Speed and Direction
- Optical Backscatter (C)
- Water Samples
Fig. 3. Laboratory calibration for an OBS (not used during AmasSeds) showing good agreement for the power law form of the curve for high concentrations.
Effects, Issues, Challenges

• Simplest case:
  \[ w_s C = -K_s \frac{\partial C}{\partial z} \]

• \( w_s \) flocs, hindered settling
• \( K_s \) = eddy diffusivity
  \[ K_s = K(\text{stratification, } Ri \text{ correction}), \]

Would like to know how stratification changes through the water column, with time.
Autonomous Profiler

- Submersible programmable winch with buoyant instrument package
- Seabird Seacat with Optical Backscatterance Sensor
- Hourly profiles
High Concentration Suspensions
Rouse Equation: Vertical Distribution of Suspended Sediments

\[
\frac{C_z}{C_a} = \left( \frac{h - z}{z} \cdot \frac{z_a}{h - z_a} \right) \frac{w_s}{\kappa u_*}
\]

\( C_z = \) conc at ht above bed \( z \)
\( C_a = \) conc at ht above bed \( a \)
\( h = \) total water depth
\( w_s = \) settling velocity
\( u_* = \) shear velocity
\( \kappa = \) von karman constant, 0.41

Geyer 1993
SSC distribution and flux based on Rouse equation (steady, horizontally uniform)

\[
\frac{w_s}{\kappa u_*} < 0.8
\]

\[
q_s = \int u_z c_z \, dz
\]

These profiles don’t look like law of the wall or Rouse.
Kirby and Parker 1983
Fig. 20. (A) Defining sketch of fluid-mud profile showing instantaneous suspended-sediment concentration and velocity. The values are intended to be representative of field observations (taken from Ross and Mehta, 1989). (B) Suspended-sediment concentration and velocity profiles through fluid mud on the Amazon shelf during high discharge at OS2 are very similar to the idealized sketch. The maximum flux, $U_{SSC}$, is within the fluid mud.

Kineke et al. 1996
• Fluid muds are dense, nearbed suspensions of fine sediments O(10 g/l) or more
  Characterized by one or more strong density gradients due to suspended sediment, lutoclines
  Alter the characteristics of the flow
Formation?

Resuspension by currents?

OR

Resuspension by waves?
Probably not resuspension by waves or currents!
Enhanced settling flux, flocculation and hindered settling are key
Ross and Mehta, 1989

Winterwerp and van Kesteren 2004

Fig. 5.3: Variation of settling velocity with data – verification of hindered settling formula (5.17).
Fluid muds first observed in estuaries, found to be thick and persistent on Amazon shelf during AMASSEDS (‘89-’91)

Kineke et al. 1996

Fig. 2. Distribution of fluid mud during the four AmasSeds cruises. Stations shaded are those with SSC > 10 g l⁻¹. Broken lines in the shading represent regions where fluid mud was not observed during the hydrographic survey, but was observed during a subsequent leg of the same cruise. The approximate location of the 24 ppt bottom salinity contour is superimposed, indicating the middle of the bottom salinity front.
Region of fluid muds does not coincide with region of turbid surface plume

Kineke et al. 1995
Open Shelf Transect
June 1990

Thanks Geyer and WHOI Graphics
Kineke et al. 1996
Motivation: Results from AMASSEDS

$$\frac{U_{\infty}}{\sqrt{B}} > \text{or} < 2?$$

B is integrated buoyancy anomaly

Trowbridge and Kineke 1994

- See also Wright, Friedrichs, Kim and Scully 2001 and following papers for application
The Petitcodiac River
The Petitcodiac River

Strong flows! High concentrations! Fluid muds!

Early Flood

~ Slack High
Objectives

• Examine controlling factors in the formation and destruction of fluid mud under a sheared flow
• Verify critical gradient Richardson number dependence for suppression of turbulence and carrying capacity of a high-concentration suspension
• Evaluate effects of mixed grain size on the settling of high-concentration suspensions
Instrumentation

- SUBS profiler

\[ \frac{du}{dz}, \rho_z (S, T, C), \text{samples} \]
Instrumentation

- ADV on gafanhoto $u', w'$
- Surfboard (ADCP, Knudsen) $U_z$, thickness
Preliminary* Results

\[ \frac{U_\infty}{\sqrt{B}} < \text{or} > 2? \]

AMASSEDS

*Caution! Kineke AGU results. Careful Masters students results in progress (Kristy Heath) show more examples below threshold
\[
\frac{U_\infty}{\sqrt{B}} < 2
\]
Appendix:

• Sediment Trapping in Winyah Bay due to Frontogenesis
Motivation:
Localized areas of high susp. sed. concentration in water column and fine sediment accumulation well downstream of turbidity max

Winyah Bay
70 km N. of Charleston
Tides ~ 1.5-2.0 m
Discharge: 425 m³s⁻¹
Motivation:
Suspended sediment concentrations often uncorrelated with boundary shear stresses
Frontogenesis
(from Geyer, pers. comm.)

Pressure Gradient:

\[
\frac{\partial p}{\partial x} = g \int_z^n \frac{\partial \rho}{\partial x} \, \partial z - g \rho_s \frac{\partial \eta}{\partial x}
\]

3 Contributing Factors:

– Baroclinic Effect
– Adverse Pressure Gradient
– Stratification
Conditions Necessary for Frontogenesis

1) Strong, but not too strong, tidal flow

\[ 1 \leq \frac{U_T}{\left( \frac{\Delta \rho}{\rho} gh \right)^{\frac{1}{2}}} < 2 \]
Conditions Necessary for Frontogenesis

2) Strong enough expansion for adverse pressure gradient

\[ \frac{\partial U_T}{\partial x} / \omega > 1 \]

\( \omega = \text{tidal frequency} \)
Conditions Necessary for Frontogenesis

3) Non-trivial baroclinic pressure gradient

\[
\frac{g}{\rho} \frac{\partial \rho}{\partial x} h > 0.2 \\
\frac{\rho}{\omega U} \frac{\partial x}{\partial x}
\]
\[ 1 \leq \frac{U_T}{\left( \frac{\Delta \rho}{\rho} \frac{gh}{\omega} \right)^{1/2}} < 2 \quad (1.3, 1) \]

\[ \frac{\partial U_T}{\partial x} > 1 \quad (2) \]

\[ \frac{g}{\rho} \frac{\partial \rho}{\partial x} \frac{h}{\omega U} > 0.2 \quad (~0.5) \]
Salinity (psu)  SSC (g/l)

max ebb

decel ebb

early flood

Trapping Length

\[ L = \frac{uh}{w_f} \]

\[ u = 30 \text{cm/s} \]
\[ h = 4 \text{m} \]
\[ w_f = 0.1 \text{cm/s} \]
\[ L = 1.2 \text{ km} \]
Observations in 2000
- time series
- floc size
- Kasten core, $^{234}\text{Th}$
Clark Alexander, SkIO

Winyah Bay 2000

$^7\text{Be}$ Activity (dpm g$^{-1}$)

$^{234}\text{Th}$ Activity (dpm g$^{-1}$)

40 cm deposition w/in 3 months

Clark Alexander, SkIO
Conclusions

• Frontogenesis is an efficient and possibly common mechanism for trapping of fine sediments (stratification + front + flocs)

• Implications for dredging operations are that dredging can enhance sedimentation, but perhaps could be minimized
SOME REFERENCES


